

Huge entropy production inside black holes

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ABSTRACT: We show that the entropy created by Ohmic dissipation inside an accreting charged black hole may exceed the Bekenstein-Hawking entropy by a large factor. If the black hole subsequently evaporates, radiating only the Bekenstein-Hawking entropy, then the black hole appears to destroy entropy, violating the second law of thermodynamics. A companion paper discusses the implications of this startling result. Bousso's covariant entropy bound is not violated.

KEYWORDS: Black Holes, Black Holes in String Theory.

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1. Introduction

The purpose of this paper is to show that classical processes of dissipation can generate huge quantities of entropy inside the horizon of a black hole, many orders of magnitude more than the Bekenstein-Hawking [1] entropy. The specific black hole model presented is intended to be semi-realistic, albeit over-simplified, with parameters appropriate to a real supermassive black hole. We take charge as a surrogate for angular momentum, and electrical conductivity as a surrogate for angular momentum transport. To see how much entropy might be created, we treat the electrical conductivity as an adjustable free parameter.

If a black hole creates many times the Bekenstein-Hawking entropy and subsequently evaporates, radiating only the Bekenstein-Hawking entropy and leaving no remnant, then entropy is destroyed, violating the second law of thermodynamics. The implications of this startling result are discussed in a companion paper [2].

Throughout this paper we treat entropy in a purely classical fashion. In particular, we assume that locality holds inside the black hole. Locality, the quantum field theory proposition that operators commute at spacelike-separated points, is the assumption that normally makes it legitimate to add entropy over spacelike surfaces. Since the spacetime curvature inside a supermassive black hole is well below Planck, except near the singularity, one might expect classical physics to apply.

It is widely thought that in order to preserve unitarity of black hole evaporation, locality must break down over spacelike surfaces connecting the inside and outside of a black hole [3]. In the companion paper [2] we argue that the calculations of the present paper point to a profligate breakdown of locality inside black holes.

We work in Planck units, $k_B = c = G = \hbar = 1$.

2. Model of entropy production inside a black hole

Real supermassive black holes acquire most of their mass not during a single collapse event, but rather by gradual accretion of gas over millions or billions of years. We model this gradual growth by the general relativistic, self-similar, accreting, spherical, charged black hole solutions described by [4, 5], to which the reader is referred for more detail. In these models the black hole accretes a charged “baryonic” plasma of relativistic matter, with constant proper pressure-to-density ratio $p/\rho = w$, at a constant rate, so that the mass of the black hole increases linearly with time.

Real supermassive black holes probably rotate, but have tiny electric charge. However, the interior structure of a spherical charged black hole resembles that of a rotating black hole in that the negative pressure (tension) of a radial electric field produces an effective gravitational repulsion analogous to the centrifugal repulsion inside a rotating black hole. Thus we follow the common practice of taking charge as a surrogate for spin. In the self-similar solutions, the charge of the black hole is produced self-consistently by the accumulation of the charge of the accreted plasma.

Similarly, we take electrical conduction as a surrogate for the dissipative transport of angular momentum. We do not use a realistic electrical conductivity, but rather treat it as a phenomenological adjustable quantity. In diffusive electrical conduction, the electric field $E = Q/r^2$ gives rise to an electric current $j = \sigma E$. If the conductivity σ is taken to be a function only of the plasma density ρ , then the condition of self-similarity forces [4]

$$\sigma = \kappa \rho^{1/2} / (4\pi)^{1/2} \quad (2.1)$$

where the dimensionless conductivity coefficient κ is a phenomenological constant. As discussed by [4], this phenomenological conductivity is greatly suppressed compared to any realistic conductivity (except perhaps at densities approaching the Planck density). However, angular momentum transport is intrinsically a much weaker process than electrical conduction, so it is not unreasonable to consider a greatly suppressed conductivity.

Since information can propagate only inwards inside a black hole (at least classically), it is natural to impose boundary conditions outside the black hole. We assume that the boundary conditions of the accreting black hole are established at a sonic point outside the horizon, where the infalling plasma accelerates from subsonic to supersonic. We assume that the acceleration through the sonic point is finite and differentiable, which imposes two boundary conditions. The accretion in real black holes is likely to be much more complicated, but this assumption is the simplest physically reasonable one.

We define the charge Q_\bullet and mass M_\bullet of the black hole at any instant to be those that would be measured by a distant observer if there were no charge or mass outside the sonic point,

$$Q_\bullet = Q \text{ and } M_\bullet = M + \frac{Q^2}{2r} \text{ at the sonic point} \quad (2.2)$$

where r is the circumferential radius, and Q and M denote the interior charge and mass, all gauge-invariant scalar quantities. If the black hole ceases accreting abruptly at some time, then Q_\bullet and M_\bullet are the actual charge and mass of the black hole at that time.

Given the assumption that the sonic point is regular, the dimensionless free parameters of the solutions are: (1) the mass accretion rate \dot{M}_\bullet ; (2) the charge-to-mass ratio Q_\bullet/M_\bullet of the black hole; (3) the equation of state parameter w ; and (4) the conductivity coefficient κ .

The black hole mass increases linearly with time, $M_\bullet \propto t$, and the mass accretion rate \dot{M}_\bullet is

$$\dot{M}_\bullet \equiv dM_\bullet/dt = M_\bullet/t, \quad (2.3)$$

where $t = \tau_d = (r\xi_d^t)_{r=r_s}$ is the time measured by clocks attached to neutral dust (d) that free-falls radially through the sonic point $r = r_s$ from zero velocity at infinity, and which therefore records the proper time at rest at infinity, and ξ_d^t is the time component of the homothetic vector ξ^k in the dust frame [6, Appendix E].

The density ρ and temperature T of a relativistic fluid in thermodynamic equilibrium are related by $\rho = (\pi^2 g/30)T^4$, where $g = g_B + \frac{7}{8}g_F$ is the effective number of relativistic particle species, with g_B and g_F being the number of bosonic and fermionic species. If the expected increase in g with temperature T is modeled (so as not to spoil self-similarity) as a weak power law $g/g_p = T^\varepsilon$, with g_p the effective number of relativistic species at the Planck temperature, then the relation between density ρ and temperature T is

$$\rho = (\pi^2 g_p/30)T^{(1+w)/w}, \quad (2.4)$$

with equation of state parameter $w = 1/(3 + \varepsilon)$ slightly less than the standard relativistic value $w = 1/3$. We fix g_p by setting the number of relativistic particles species to $g = 5.5$ at $T = 10 \text{ MeV}$, corresponding to a plasma of relativistic photons, electrons, and positrons.

The entropy S of a proper Lagrangian volume element V of an ideal relativistic fluid with zero chemical potential is $S = [(\rho + p)/T]V$. The proper velocity of the baryonic fluid through the sonic point equals the ratio ξ^r/ξ^t of the radial and time components of the homothetic vector in the plasma frame [4]. Thus the entropy S accreted through the sonic point per unit proper time of the fluid is $dS/d\tau = [(1+w)\rho/T]4\pi r^2(\xi^r/\xi^t)$. The sonic radius r_s of the black hole increases as $d \ln r_s/d\tau = 1/(r\xi^t)_{r=r_s}$ [4]. The Bekenstein-Hawking entropy of the black hole is $S_{\text{BH}} = \pi r_h^2$ where r_h is the horizon radius, so $dS_{\text{BH}}/d \ln r_s = 2\pi r_h^2$. Thus the entropy S accreted per unit increase of the Bekenstein-Hawking entropy is

$$\frac{dS}{dS_{\text{BH}}} = \frac{1}{2\pi r_h^2} \left. \frac{(1+w)\rho 4\pi r^3 \xi^r}{T} \right|_{r=r_s}. \quad (2.5)$$

Inside the sonic point, dissipation increases the entropy. The energy-momentum tensor is the sum of baryonic and electromagnetic parts, T_b^{mn} and T_e^{mn} , and the evolution of baryon entropy is determined by the time component of the equation of covariant conservation of energy-momentum in the rest frame of the baryons:

$$D_m T_b^{mt} = -D_m T_e^{mt}. \quad (2.6)$$

In the self-similar model being considered, the energy conservation equation (2.6) can be written [4]

$$\frac{d\rho}{d\tau} + (1+w)\rho \frac{d\ln(r^3\xi^r)}{d\tau} = \sigma \frac{Q^2}{r^4} \quad (2.7)$$

which can be recognized as an expression of the first law of thermodynamics $d\rho V + p dV = T dS$ with proper volume $V \propto r^3 \xi^r$. The right hand side of equation (2.7) is the Ohmic dissipation $jE = \sigma E^2$. Equation (2.7) can be re-expressed as

$$\frac{d\ln S}{d\tau} = \frac{\sigma Q^2}{(1+w)\rho r^4} \quad (2.8)$$

with $S \propto \rho^{1/(1+w)} r^3 \xi^r \propto (\rho/T) r^3 \xi^r$.

Since other physics presumably takes over near the Planck scale, we truncate the production of entropy at some arbitrary maximum density $\rho_\#$ (“rho splat”). Integrating equation (2.8) from the sonic point to the splat point yields the ratio of the entropies at the sonic and splat points. Multiplying the accreted entropy, equation (2.5), by this ratio yields the rate of increase of the entropy of the black hole, truncated at the splat point, per unit increase of its Bekenstein-Hawking entropy

$$\frac{dS}{dS_{\text{BH}}} = \frac{1}{2\pi r_h^2} \left. \frac{(1+w)\rho 4\pi r^3 \xi^r}{T} \right|_{\rho=\rho_\#} . \quad (2.9)$$

The entropy created becomes large when the conductivity coefficient lies in the range $\kappa \approx 1.3$ to 1000. Over this range the rate dS/dS_{BH} of increase of entropy, equation (2.9), is almost independent of the black hole mass M_\bullet ,

$$\frac{dS}{dS_{\text{BH}}} \approx \text{const} \approx \frac{2(1 - Q_\bullet^2/M_\bullet^2)^{1/2}(1+w)\rho_\#}{\dot{M}_\bullet [1 + (1 - Q_\bullet^2/M_\bullet^2)^{1/2}]^2 \sigma_\# T_\#} \quad (2.10)$$

in which the empirical fit on the right hand side is accurate to a factor of two over the range $\kappa \approx 10$ to 1000 (for $\kappa \lesssim 10$ to ≈ 1.3 , the fit increasingly overestimates dS/dS_{BH}), and $M_\bullet \gtrsim 3 M_\odot$, $\dot{M}_\bullet \lesssim 10^{-4}$, $Q_\bullet/M_\bullet \approx 10^{-12}$ to 0.99999, $w \approx 0.1$ to 0.55, and $\rho_\#$ not too small.

Bousso [7] has proposed the covariant entropy bound, which states that the entropy passed through a converging lightsheet cannot exceed $S_{\text{cov}} \equiv \text{area}/4$ of its boundary. In the models under consideration, the rate at which entropy passes through an ingoing or outgoing spherical lightsheet per unit decrease in $S_{\text{cov}} = \pi r^2$ is

$$\left| \frac{dS}{dS_{\text{cov}}} \right| = \frac{dS}{dS_{\text{BH}}} \frac{r_h^2}{r^2} \frac{1}{\xi^r |\beta \mp \gamma|} \quad (2.11)$$

in which $\{\beta, \gamma\}$ are the time and radial components of the proper covariant radial 4-gradient in the notation of [4], and the \mp sign is $-$ for ingoing, $+$ for outgoing lightsheets. A sufficient condition for the covariant entropy bound to be satisfied is $|dS/dS_{\text{cov}}| \leq 1$.

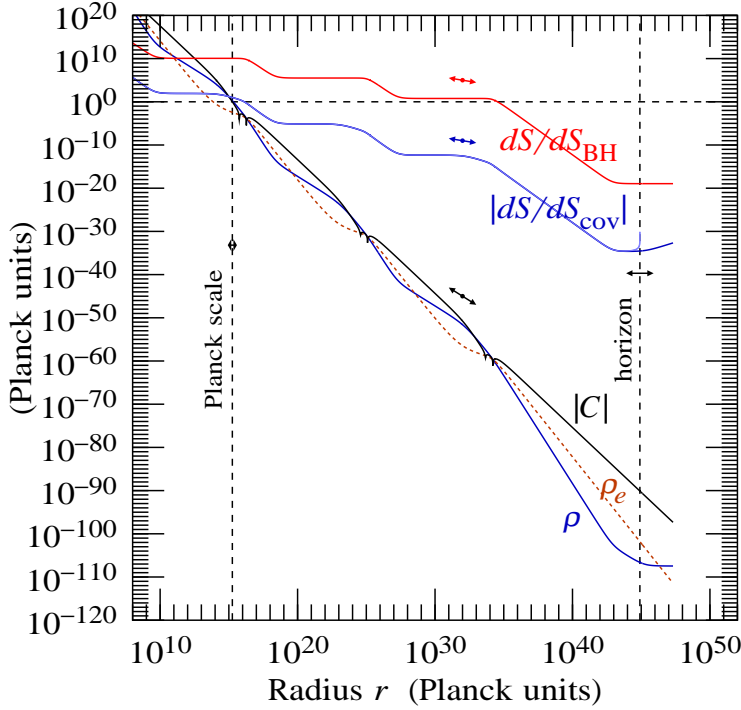


Figure 1: (Color online) Internal structure of a black hole with mass $M_{\bullet} = 4 \times 10^6 M_{\odot}$, accretion rate $\dot{M}_{\bullet} = 10^{-16}$, charge to mass $Q_{\bullet}/M_{\bullet} = 10^{-5}$, equation of state $w = 0.32$, and conductivity coefficient $\kappa = 1.24$. The quantities plotted are, as a function of radius r : the density ρ of the baryonic plasma, the energy density ρ_e (short dashed line) of the static electric field, the absolute value of the Weyl curvature scalar $C = 4\pi\rho/3 + Q^2/(2r^4) - M/r^3$, the rate dS/dS_{BH} of increase of the black hole entropy with Bekenstein-Hawking entropy, equation (2.9), and the rate $|dS/dS_{\text{cov}}|$ at which entropy passes through ingoing (dark) and outgoing (light) spherical lightsheets per unit decrease in their area/4, equation (2.11). Vertical dashed lines mark the horizon, and where the Weyl curvature $|C|$ exceeds 1 Planck unit. Arrows, such as that above dS/dS_{BH} , show how the curves shift a factor of ten into the past and the future. The rate dS/dS_{BH} is almost independent of the black hole mass M_{\bullet} , at fixed splat density $\rho_{\#}$.

3. Example of interior entropy exceeding Bekenstein-Hawking

A black hole of mass $4 \times 10^6 M_{\odot} \approx 4 \times 10^{44}$ Planck units (the mass of the supermassive black hole at the center of the Milky Way [8, 9]) accreting over the age of the Universe $10^{10} \text{ yr} \approx 6 \times 10^{60}$ Planck units has an accretion rate of $\dot{M}_{\bullet} \approx 10^{-16}$. Figure 1 shows the interior structure of a black hole with that mass and accretion rate, charge-to-mass $Q_{\bullet}/M_{\bullet} = 10^{-5}$, equation of state $w = 0.32$, and conductivity coefficient $\kappa = 1.24$.

To produce lots of entropy, the baryonic plasma must fall to a central singularity, and we choose the conductivity $\kappa = 1.24$ to be at (within numerical accuracy) the critical conductivity [4] for this to occur. Below the critical conductivity the plasma generally does not fall to a singularity, but rather drops through the Cauchy horizon. The latter solutions are subject to

the mass inflation instability [10, 5], a fascinating regime not considered in this paper.

Solutions at the critical conductivity exhibit [4] the periodic self-similar behavior first discovered by [11]. The ringing of the curves shown in Figure 1 is a manifestation of this, not a numerical error.

The electric charge advected by the plasma inwards across the sonic point is, thanks to the “high” conductivity, almost canceled by an outward current. As a consequence, the charge-to-energy of the accreted plasma, here ≈ 400 at the sonic point, is substantially larger than the charge-to-mass of the black hole. We have deliberately chosen a small charge-to-mass ratio for the black hole, $Q_\bullet/M_\bullet = 10^{-5}$, so that the Lorentz repulsion of the plasma by the black hole is subdominant, and the trajectories of parcels of plasma outside the black hole are not greatly different from Schwarzschild geodesics. Thus for example the sonic point is at a radius of 3.06 geometric units ($c = G = M_\bullet = 1$), close to that expected for a neutral tracer relativistic fluid that free-falls from zero velocity at infinity. The horizon is at 2.00 geometric units, like Schwarzschild. Figure 1 shows the solution out to 2,000 geometric units.

At the sonic point, the plasma temperature is $\approx 4 \times 10^5$ K. Inside the horizon, the electric field increases, and Ohmic dissipation starts to heat the plasma, increasing its temperature and entropy. When the plasma energy has become comparable to the electric energy, then the plasma goes into a power law regime where the plasma and electric energies increase in proportion to each other, kept in lockstep by the conductivity.

The entropy hits the Bekenstein-Hawking milestone, $dS/dS_{\text{BH}} = 1$, when the temperature is $\approx 3 \times 10^{-16}$ Planck units, or 3 TeV, and the curvature radius is $|C|^{-1/2} \approx 10^{30}$ Planck lengths, or 0.01 mm. This temperature and curvature are almost independent of the mass M_\bullet of the black hole, equation (2.10).

If the plasma’s dissipative trajectory is followed to the Planck scale, $|C| = 1$, then the rate of increase of entropy relative to Bekenstein-Hawking is $dS/dS_{\text{BH}} \approx 10^{10}$, again almost independent of the mass M_\bullet of the black hole. If the entropy is assumed to accumulate additively inside the black hole then the cumulative entropy can evidently exceed Bekenstein-Hawking by a large factor.

Figure 1 shows that $|dS/dS_{\text{cov}}| \leq 1$, equation (2.11), at all sub-Planck scales. Thus although the cumulative entropy may exceed Bekenstein-Hawking, Bousso’s covariant entropy bound is satisfied by the black hole.

4. Conclusion

We have shown that the dissipation of the free energy of the electric field inside a charged black hole can potentially create many times more than the Bekenstein-Hawking entropy. If the black hole subsequently evaporates, radiating only the Bekenstein-Hawking entropy and leaving no remnant, then entropy is destroyed.

This startling conclusion is premised on the assumption that entropy created inside a black hole accumulates additively on spacelike slices, which in turn derives from the assumption that the Hilbert space of states is multiplicative over spacelike-separated regions, as

postulated by locality. This is essentially the same reasoning that originally led Hawking [12] to conclude that black hole evaporation is non-unitary, that black holes must destroy information.

It is widely thought that unitarity should be considered a higher principle than locality. To ensure that black hole evaporation is unitary, locality between the inside and outside of a black hole must break down. In a companion paper [2] we argue that the gross violation of the second law found in the present paper points to a wholesale breakdown of locality inside black holes, and provides a compelling argument in favor of the conjecture of “observer complementarity”.

The black hole respects Bousso’s covariant entropy bound [7], as it should given the theorem of [13].

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